**LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034**

**M.Sc.** DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – NOVEMBER 2015

**ST 3816 - STOCHASTIC PROCESS**

Date : 05/11/2015 Dept. No. Max. : 100 Marks
Time : 09:00-12:00

## SECTION - A

**Answer ALL the questions.** **(10 x 2 = 20 marks)**

1. Define a process with Independent increments.
2. Define a Markov chain.
3. Obtain the period of state 0 in the Markov chain with the Transition Probability Matrix with states 0,1,2. $P=\begin{pmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{pmatrix}$
4. Write the pdf of the inter arrival time T for a Poisson process with parameter λ.
5. If a Markov chain is recurrent, irreducible and aperiodic then the basic limit theorem gives $\lim_{n\to\infty} P_{ii}^{n} =$ ______
6. Define renewal process.
7. When do you say the process $\{X_n\}$is a Martingale with respect to the process $\{Y_n\}$?
8. Given the Markov chain with states 0,1 and Transition Probability Matrix $P=\begin{pmatrix} 1 & 0 \\ 1/2 & 1/2 \end{pmatrix}$. Identify whether the states are recurrent or transient.
9. Explain branching process.
10. Define current life and excess life.

## SECTION - B

**Answer any FIVE questions.** **(5 x 8= 40 marks)**

11. Explain the classification of stochastic process with examples.
12. Consider the Markov chain with states 0,1,2 and Transition Probability Matrix $P=\begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$

    and the initial distribution $P[X_0 = i] = \frac{1}{3}$ i=0,1,2.

    Obtain i) $P[X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2]$

    ii) $P[X_3 = 2, X_2 = 1 | X_1 = 0, X_0 = 1]$

    iii) $P[X_2 = 2]$ **(2+2+4)**
13. Write the postulates of pure death process and obtain the solution of the differential equations using «$_n$ = $n$« .

14. Show that $X_n = (\sum_{1}^{n} Y_k)^2 - n\pm^2$ where $Y_k$ are iid, $E[Y_k] = 0,\ V[Y_k] = \pm^2, Y_0 = 0,$ is a Martingale.

15. In a two state birth and death process, the communication is between only two states 0 and 1. With the appropriate postulates and infinitesimal matrix obtain $P_{00}(t)$.

16. Obtain the mean and variance of branching process.

17. Show that a state i is recurrent if and only if $\sum_{n} P_{ii}^{\ n} = \infty$.

18. Obtain the probability of absorption into state 0 in a gamblers ruin problem, using first step analysis.

## SECTION - C

**Answer any TWO questions.** **(2 x 20= 40 marks)**

19. a) Let P be the regular Transition Probability Matrix on the states 0,1,2,…,N. then show that the limiting distribution $(\Pi_0, \Pi_1, ..., \Pi_N)$ is the unique non-negative solution of the equations.

$$\Pi_j = \sum_k \Pi_k P_{kj} \quad and \quad \sum_k \Pi_k = 1$$

b) Consider the Markov chain with the Transition Probability Matrix

$$\begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1/3 & 2/3 & 0 & 0 \\ 1/3 & 2/3 & 0 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Obtain $\lim_{n\to\infty} P_{2i}^{n}, \quad i = 0,1,2,3$ **(12+8)**

20. a) State the postulates of Poisson process and hence obtain an expression for $P_n(t)$.

b) if $X_1(t)$ and $X_2(t)$ are independent Poisson process with parameter $a_1$ and $a_2$.

Obtain the distribution of i) $X_1(t) + X_2(t)$ ii) $X_1(t) = k \ given \ X_1(t) + X_2(t) = n$ **(12+8)**

21. a) For a renewal process obtain $E[N(t)] \ \ and \ \ E[W_{N(t)+1}]$

b) Obtain the renewal equation for a discrete time renewal $M(n) = F(n) + \sum_{k-1}^{n-1} P_k M(n-k)$

c) Obtain the distribution of the current life in the Poisson process. **(12+4+4)**

22. a) Obtain the probability generating function relations for a branching process.

b) How to obtain the probability of extinction for a branching process. Explain with examples. **(10+10)**

************